Abstract: We will discuss some basic problems in representation theory of finite groups, including some long-standing conjectures of Alperin, Brauer, and others. A possible approach to some of these problems is to use the classification of finite simple groups to reduce the problem in consideration to some, more specific, questions about simple groups. We will describe recent progress on reduction theorems in this direction. We will also outline applications of group representation theory to various problems, particularly in number theory and algebraic geometry.