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Real-valued characters of finite reductive groups

The main topic of this talk will be a result which is joint work with Bhama Srinivasan (U. Illinois–Chicago). Let $G$ be the group of $F_q$-points of a connected reductive group with connected center defined over a finite field $F_q$ with $q$ elements. Let $\chi$ be an irreducible complex character of $G$, with Jordan decomposition (Lusztig parameters) given by $(s, \psi)$, with $s$ semisimple in the dual group $G^*$, and $\psi$ a unipotent character of its centralizer. We prove that $\chi$ is real-valued if and only if $(s, \psi)$ is $G^*$-conjugate to $(s^{-1}, \bar{\psi})$. The main tool is a result of Digne and Michel which describes the Jordan decomposition map as being unique with respect to a list of 8 properties. Further, we will discuss possible extensions of this method to understand the action under an arbitrary Galois element, and evaluating Frobenius-Schur indicators.