The automorphism groups of the random graph and its relatives
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For a fixed positive integer $n$, if we colour the edges of the complete graph on a countable vertex set randomly with $n$ colours, there is a particular configuration which arises with probability 1. Its group $G$ of strict (colour-preserving) automorphisms is simple, and the outer automorphism group of $G$ is $S_n$ (corresponding to permutations of the colours). The automorphism group of $G$ splits over $G$ if and only if $n$ is odd. If $n$ is even and not a multiple of 8, there is a finite supplement to $G$; this is unknown in the remaining case, where it reduces to a problem about finite groups.

In the case $n = 2$, the configuration consists of the celebrated random graph and its complement. I will also talk about embeddings of its automorphism group as a group of automorphisms of a filter, or homeomorphisms of a topology, on the vertex set.