Counting prime order subgroups of finite groups

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Let $G$ be a finite group, let $i_2(G)$ be the number of involutions in $G$ and let $\delta(G)$ be the number of prime order subgroups of $G$. In 1970, C.T.C. Wall classified the finite groups $G$ with $i_2(G) > |G|/2 - 1$. In recent work with Stuart Scott, we extend Walls theorem by determining the groups $G$ with $\delta(G) > |G|/2 - 1$. The proof uses the Classification of Finite Simple Groups, and some interesting results are obtained along the way. I will describe our motivations for studying this problem and I will explain the main steps in the proof. I will also outline an application of the main theorem to the prime generation problem in finite near-rings, and discuss some related results and open problems.