Definability in algebraically closed valued fields
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Algebraically closed valued fields arise naturally, for example through Puiseux series, or through the algebraic closure of the $p$-adics and its completion. Already in the 1950s, Abraham Robinson showed that the model theory of such structures is tractable: the first-order definable sets have a rather simple form, namely, boolean combinations of sets defined by polynomial expressions $f(x) = 0$ and $v(f(x)) < v(g(x))$.

I will give an overview of a body of work, beginning with an adaptation for valued fields of the notion of o-minimality (work with Haskell and Steinhorn), and leading to a coding of definable sets by lattices (and other modules) over the valuation ring (with Haskell and Hrushovski). I hope to sketch some more recent developments: an application of such ideas to prove rationality of a power series associated with representations of finitely generated nilpotent groups (Hrushovski-Martin), and recent work of Hrushovski and Kazhdan related to motivic integration.